Physical and Economic Design Criteria for Induced Snow Accumulation Projects[1]

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Abstract. Snow fencing promises to be an important means of increasing surface water yield or ground water recharge on windswept watersheds where snow is an important form of precipitation. Assuming an equally spaced series of snow fences, a physical production function can be developed that relates fence spacing to the 'most probable' annual snow catch, based on a probability analysis of winter precipitation. The optimum scale of development in terms of fence spacing, determined by standard marginal analysis, indicates that the smaller the marginal value of output with respect to inputs, the greater the probability must be of the fences filling annually if net benefits are to be maximized. (Key words: Economic analysis; snow fences; snow)

INTRODUCTION

The increasing demand for water has stimulated investigations of snowpack management as a possible means of regulating or increasing water yield in the western United States. Analysis of hydrologic data indicates that watershed management in the West can be most productive in the snowpack zone [*Select Committee,* 1960].

Most snow management research has been directed toward relationships between vegetative cover and snow disposition. However, artificially induced drifting of snow on a watershed may also increase surface water yield or ground water recharge. The hypothesis of increased yield from this form of snowpack management is based on the assumption that significant evaporative losses occur during prolonged transport of snow by wind, and that these losses can be substantially reduced by retaining the snow where it falls. The hypothesis of a regional net benefit may be predicated, in addition, on the value of timing runoff from snowmelt, or on the assumption that the snow loses value when it is blown from one area and deposited in another. This latter proposition is not unique to snowfencing, for it has an equivalent in all water-diversion projects: water may be of greater value in one drainage basin than in another.

Snow management problems that must be resolved by physical and economic considerations are demonstrated by essential design considerations for a snow fence project: (1) Type and height of fence; (2) Arrangement of fences on an area (Should drifting be induced only in specific locations, or should fences be regularly spaced over an area?); (3) Snow retention capacity for which a project should be designed, given (1) and (2). Additional research is needed for requisite knowledge of the physical aspects of snow transport phenomena. Economic analysis can then guide practical application, as demonstrated in this paper.

THE PHYSICAL MODEL

The problem. The model for this study is based on long rows of stationary snow fences of a given type and height at right angles to the prevailing wind direction during drifting events on a watershed. The distance between rows (*spacing*) is assumed to be uniform, implying the absence of natural barriers to wind movement over a relatively flat or gently rolling topography. It is further assumed that the

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project is proposed for a suitable area, as defined by the following physical characteristics: (1) Snow is a significant source of water yield from the area; (2) Conditions are conducive to snow drifting during part of the winter [*Landsberg,* 1947]; (a) Air temperatures below freezing are common; (b) Snow exists in a condition conducive to transport by wind; (c) Wind velocities of 10 mph, or greater, are common; (3) Snow retention on the area can be increased significantly.

The objective of this paper is to investigate the physical and economic relationships that will determine the spacing of snow fences on an area and thus provide a means of determining the snow retention capacity for which a project should be designed.

Physical theory. The saturation capacity for a snow fence of given design and height (the amount of snow it will contain when filled) can be considered a constant that is independent of wind velocity and the amount of snow entering the fence domain [*Finney,* 1934; *Pugh,* 1950; *Pugh and Price,* 1954]. The fundamental cross-sectional shape of a drift is characterized by an upper surface resembling an ichthyoid curve, with maximum height equal to height of the fence [*Pugh,* 1950]. It is generally recognized that only two variables influence the length of a snow fence drift on a given site: fence height and density (density is the per cent of solid area of a slatted fence). An equation derived by German investigators [*Bekker,* 1951] is an example

$$L = [360(7.22 + h)]/(22.1 + d) \qquad (1)$$

where

L = drift length (feet);
h = fence height (feet);
d = fence density (per cent).

The cross-sectional area of a drift is thus related to the square of the fence height, since fence density remains constant [*Pugh,* 1950]. Effects of wind amelioration apparently are not cumulative in a series of fences if the distance between fences is 40 or more times fence height [*Gloyne,* 1957].

The preceding relationships make it possible to estimate the amount of snow (or inches water-equivalent) required to saturate a fence of given height and density. In this study, optimum height and density of a fence were selected as a starting point. The mechanics of snow transport suggest a limit to the fence height-drift volume relationship described. It remains to be determined, however, whether such a restriction is of more importance than the economic limitation of construction cost. It is generally agreed that the optimum density for an open fence is approximately 50% [*Pugh and Price,* 1954].

If we consider a series of very long fences of a given design constructed over an infinite distance downwind, and assume a 100% collection efficiency for each fence, the spacing required so that the contributing area of each fence would provide just sufficient snow during an average

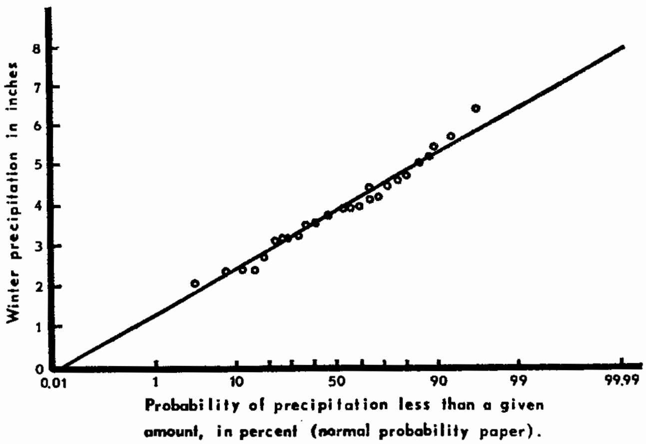

Fig. 1. Probability distribution of winter precipitation (November through April) for an area near Laramie, Wyoming (26 years of record), with average value of 3.91 inches.

TABLE 1. Computations for a Physical Production Schedule Relating Number of Fences per Mile to the Most Probable Annual Catch

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Precip. Interval Limit X_L, in.	Fence Spacing S, ft	Midpoint of Precip. Interval $\bar{X}_i$, in.	Prob. of Precip. $< X_L$ $100F(X_L)$, %	Frequency p_i, %	Weighted Precip. $p_i\bar{X}_i$, in.	Accum. Col. (6) $\sum p_i\bar{X}_i$, in.	Prob. of Precip. $> X_L$ $100[1 - F(X_L)] = 100 - \sum p_i$, %	Weighted Precip. $X_L(100 - \sum p_i)$, in.	Average Annual Catch $[(7) + (9)]/100$ in.	Average Annual Catch, acre − ft/mi²/yr	Fences per Mile, no.
0.00	...		0.0			0.00	100.0	0.00	0.000	...	...
		0.25		0.1	0 02						
0.50	1264		0.1			0.02	99.9	49.95	0.500	26.67	4
		0.75		0.3	0.22						
1.00	632		0.4			0.24	99.6	99.6	0.998	53.23	8
		1.25		1.0	1.25						
1.50	421		1.4			1.49	98.6	147.9	1.494	79.68	13
		1.75		2.6	4.55						
2.00	316		4.0			6.04	96.0	192.0	1.930	105.60	17
		2.25		6.0	13.50						
2.50	253		10.0			19.54	90.0	225.0	2.445	130.40	21
		2.75		10.0	27.50						
3.00	211		20.0			47.04	80.0	240.0	2.870	153.07	25
		3.25		14.6	47.45						
3.50	181		34.6	18.4	69.00	94.49	65.4	228.9	3.234	172.48	29
4.00	158	3.75				163 49	47.0	188.0	3.515	187.47	33
		4.25	53.0	17 0	72.25						
4.50	140		70.0			235.74	30.0	135.0	3.707	197.71	38
		4.75		13.5	64.12						
5.00	126		83.5			299.86	16.5	82.5	3.824	203.95	42
		5.25		8.7	45.68						
5.50	115		92.2			345.54	7.8	42.9	3.884	207.15	46
		5.75		4.8	27.60						
6.00	105		97.0			373.14	3.0	18.0	3.911	208.59	50
		6 25		2.0	12 50						
6.50	97		99.0			385.64	1.0	6.5	3.921	209.12	54
		6.75		0.7	4.72						
7.00	90		99.7			390.36	0.3	2.1	3.925	209.33	59
		7.25		0.24	1.74						
7.50	84		99.94			392.10	0.06	0.4	3.925	209.33	63
		7.75		0.05	0.39						
8.00	79		99.99			392.49	0.01	0.1	3.926	209.39	67

year to fill the fence may be derived as follows:

$$S = 12W/P \tag{2}$$

where

S = fence spacing at saturation (ft);
W = snow storage at saturation (cu ft of water per lineal ft of fence);
P = winter precipitation in the form of snow (in.).

To a reasonable approximation

$$W = \tfrac{1}{2}\rho h L \tag{3}$$

where

h = fence height (ft);
L = drift length at saturation (ft);
ρ = expected value of sample ratio of melt water volume to snow volume when fences are saturated

and subsequently

$$S = (6\rho h L)/P \tag{4}$$

If winter snowfall did not vary from year to year, the problem would be solved at this point. The question posed by variability is essentially this: should the spacing be such as to trap the median winter's precipitation (the fence thus being filled on an average of 50 years out of 100 and less than saturated an equal amount of the time), or would some other spacing result in greater net benefits?

ANALYSIS

To solve this problem, we must formulate a production function from which the point of maximum net benefits may be derived, in terms of snow fence input. The general method has been applied to many different types of economic problems [*Heady and Dillon*, 1961] and is described in most introductory texts on economic analysis. In this study, probability is used in the calculation of the physical production schedule.

Winter precipitation amounts (November through April, inclusive) for an area near Laramie, Wyoming, were plotted on normal-probability paper (Figure 1), with a probability plotting position of $m/(n + 1)$, where m is the rank of the event (l for maximum and n for minimum) and n is the number of years of record [*Subcommittee on Hydrology*, 1966]. The data in this example appear to be normally distributed as a straight-line fit. The derived distribution provides a basis for a schedule of total physical product (volume of water stored per unit area), as shown in the example of Table 1. The average annual water-equivalent catch $\bar{\bar{X}}$ for a tabular fence spacing S is a weighted average given in general by the formula

$$\bar{\bar{X}}_S = 1/N \sum_{i=1}^{k} p_i \bar{X}_i \tag{5}$$

where

p_i = frequency associated with interval mean precipitation $\bar{X}_i$;
$N = \sum p_i$;
k = number of intervals.

Fence spacing is calculated from equation 4, assuming that h = 4.5 ft (based on standard 4-ft slatted fence erected with a 0.5-ft gap beneath the fence), d = 50%, and ρ = 0.40. To

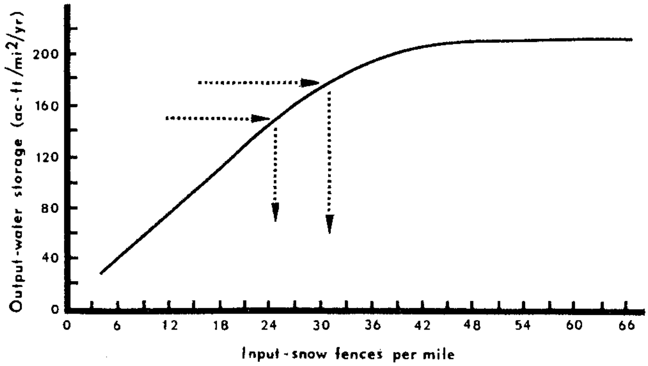

Fig. 2. Physical production function relating water storage to snow fences per mile.

account for years when the fences are less than saturated, as well as those in which they are filled, equation 5 becomes

$$\bar{X}_S = \frac{1}{N}\left[\sum_{i=1}^{k_L-1} p_i \bar{X}_i + X_L\left(100 - \sum_{i=1}^{k_L-1} p_i\right)\right] \quad (6)$$

where X_L is the interval precipitation limit associated with S.

The final result of the calculations described above and illustrated in Table 1 is a water yield production function (Figure 2), which relates number of fences per mile to the most probable annual catch (acre-ft/mi²/yr). The related marginal and average physical product curves shown in Figure 3 can be interpreted in terms of probability. Marginal and average physical productivity are at a maximum throughout the range of fence spacings where there exists a probability of 1.0 (for all practical purposes) that the fences will be saturated annually. For spacing less than the minimum exhibiting this characteristic, marginal and average productivity decline.

APPLICATION

A standard economic analysis can be used to determine the optimum scale of development for the snow fence project, once the production function has been derived.

As an example, assume for simplicity that snow fence construction is the only project variable cost, thereby ignoring maintenance costs, land acquisition, and so forth. If we use the data of Table 1, and assume a construction cost of $0.55 per lineal foot of fence, a marginal cost schedule can be obtained by dividing each tabular increase in total cost by the corresponding increase in output. The resulting marginal cost curve, Figure 4, is based on a 20-year life expectancy for the fences.

By entering the curve with a value of water applicable to the project, $35 per acre-foot, for example, the most profitable output can be determined (177 acre-ft/sq mi) and translated to snow fences per mile (31) by means of the physical production function of Figure 2.

By comparison, were storage water valued at $25 per acre-foot, the economically optimum number of fences would be reduced to about 25 per mile. This relationship indicates that the smaller the marginal value of output with respect to inputs, the greater the probability must be of the fences filling annually if net benefits are to be maximized.

Implicit in this analysis is the simplifying assumption that snow normally transported from the project area in the absence of fences has no discernible value to the project beneficiaries, regardless of the ultimate fate of the untrapped snow. To assume otherwise necessitates the formidable task of determining the *differential* value of snow water equivalent accumulated behind the fences.

Of the many questions that arise in connection with such an economic appraisal, a major consideration is that of selecting a period of analysis, that is, a length of time over which revenues and costs could be accrued for analysis. Periods of 50 to 100 years are generally

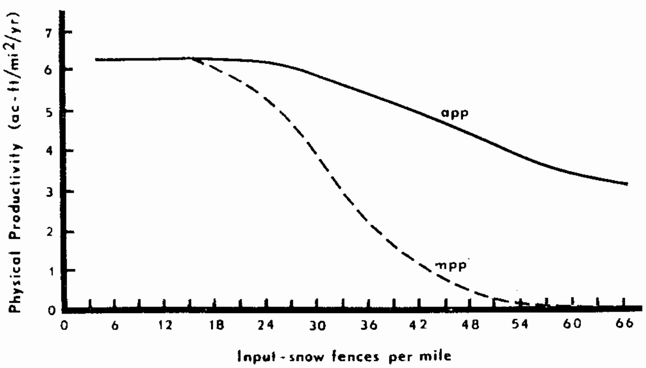

Fig. 3. Average (app) and marginal (mpp) physical productivity curves derived from the production function.

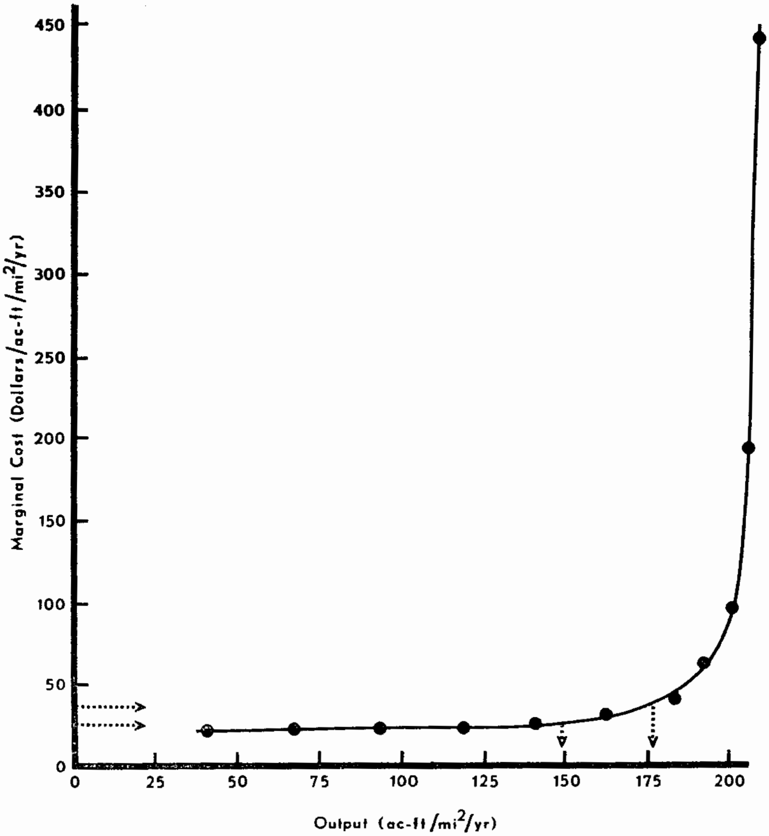

Fig. 4. Marginal cost curve based on a 20-year life expectancy for the fences.

recommended for water resource development projects with long physical and economic lives. Although comparably durable snow fences could be built at rather great expense, budgetary constraints probably would preclude this in most situations. In addition, the extreme technological uncertainty expected to be characteristic of a snow fence project would introduce a large element of risk that would be difficult to adjust for over a period of 50 years. Under these conditions, it would be desirable to construct a project with a sufficient physical life so that costs could be amortized over the period, while retaining an acceptable degree of decision flexibility. It appears reasonable to select the expected life-span of the initial structures as the period for analysis.

Although the preceding example oversimplifies the situation as it would be encountered in practice, the approach provides a basic framework that will permit elaboration in application. Implicit but unstated in the foregoing model, for example, is the fact that land area is a second variable in the problem. Scale of development can be changed by varying fence spacing or amount of land to which the treatment is applied. This consideration requires a best combination analysis involving more complex economic calculations than can be justified in this paper. *Boulding* [1955] is a recommended reference.

A second omission in the model is the possible benefit from changing the timing of water yield. Unfortunately, discussion of this dual output problem is limited by the lack of available information as to how melt rate is related to various physical factors, and what the potential would be for manipulating time or rate of melt in an applied situation. Given these data and information as to what period of retardation of snowmelt is economically significant, it will be possible to relate benefits from timing management to parameters such as number of fences per mile.

CONCLUSIONS AND SUMMARY

The analysis of an induced snow accumulation project presented in this paper is but one demonstration that economic analysis will be essential for planning and evaluating watershed management practices and comparing alternatives. The general methods proposed in this paper should be applicable to many different problems.

For an equally spaced series of snow fences, the following concepts apply:

1. Fence spacing may be calculated on the basis of winter precipitation, contributing area, and maximum snow retention capacity.

2. A probability analysis of winter precipitation provides a basis for a weighted physical production function.

3. Optimum scale of development can be determined by means of standard marginal analysis.

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